permeability of vacuum; $\bar{\chi}$, magnetic susceptibility; H_0, intensity of the external magnetic field; n_x, demagnetizing factor; S, dimensionless complex.

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DIFFUSION-CONVECTION MODEL OF GRAVITATIONAL SEPARATION IN A POLYDISPERSE SUSPENSION

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A model of the evolution of the volume concentration fields of the separate fractions of a polydisperse suspension is presented. For the case of two fractions a solution is obtained in the static regime illustrating the mechanism of separation with respect to particle size.

The majority of papers on the mathematical modeling of disperse flows in technological apparatus use a system of equations for a multivelocity continuum [1]. The hyperbolic nature of this system leads to solutions of the wave type with very sharp surfaces of discontinuity (see, for example, [2]). In practice, however, the concentration fields of the components of a disperse mixture are spread out, and this implies that the equations describing their evolution are parabolic. In [3] a parabolic equation was introduced (the Fokker–Planck equation) for the distribution functions of the velocities and positions of solid particles in a suspension, taking into account random forces of the white noise type which act on the particles. Such forces can result from the turbulence of the flow of the liquid phase [4], but, as indicated in [5], often laminar motion exists as well.

In the present paper we derive a system of equations of the parabolic type for the concentration fields of narrow fractions of a polydisperse suspension. It is assumed that the temporal viscous relaxation of the velocities of the solid particles can be neglected and that their steady-state values are determined with the help of well-known semiempirical formulas.

We write the equation of motion of a particle in the Stokes regime of sedimentation:

$$\frac{\rho_p \pi d^3}{6}\,\frac{du}{dt}=\frac{(\rho_p-\rho_f)\,\pi d^3 g}{6}+3\pi\mu d\Phi(c)\,w+F'(t), \tag{1}$$

where the dimensionless function $\Phi(c)$ takes into account the effect of the other particles (hindrance) on the hydrodynamical drag. It is assumed that the most important contribution to the stochastic term $F'(t)$ comes from fluctuations in c:

$$F'=3\pi\mu d\,\langle w\rangle\,\frac{d\Phi}{dc}\,\Delta c, \tag{2}$$

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where $\langle \Delta c(t) \Delta c(t') \rangle = \sigma^2 K(t-t')$ (the angular brackets denote an average of the quantity inside). If the decay time of the correlation function of the fluctuations is small, then one can put, approximately [6]

$$\langle \Delta c(t) \Delta c(t') \rangle = \sigma^2 T \delta(t-t'), \tag{3}$$

where $T = 1/2 \int\limits_{-\infty}^{\infty} K(t)\,dt$ and the process F'(t) is then assumed to be white noise. Equation (1) can be rewritten in the form

$$\tau^s \frac{du}{dt} = u^s(c) + w + \sqrt{2D}\,\xi(t), \tag{4}$$

where

$$\tau^s = \frac{\rho_p^{\Phi} d^2}{18\mu\Phi(c)}; \quad u^s(c) = \frac{(\rho_p - \rho_f)\, d^2 g}{18\mu\Phi(c)}; \quad D = \frac{\sigma^2 \langle w \rangle^2 T}{2} \left(\frac{d\ln\Phi}{dc}\right)^2.$$

The quantiti s τ^s and u^s are the relaxation time and the steady-state value of the sedimentation velocity in the presence of hindrance. In correspondence with the remarks made in the Introduction, we assume that $\tau^s \ll 1$ and therefore the left-hand side of (4) can be neglected. In this case we have from (4):

$$\frac{dz}{dt} \equiv u(t) = u^s(c) + v + \sqrt{2D}\,\xi(t), \tag{5}$$

and, since we obviously have $\langle w \rangle = -u^s(c)$, we obtain the following relation for the diffusion coefficient

$$D = \frac{\sigma^2 T (u^s(c))^2}{2} \left(\frac{d\ln\Phi}{dc}\right)^2. \tag{6}$$

Hence $z(t)$ is a diffusion process, and the probability density of this process can be described by the Fokker–Planck–Kolmogorov equation [6]

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial z}\left(D\frac{\partial p}{\partial z} - (u^s + v)\,p\right). \tag{7}$$

We assume now that we have n types of solid particles of different sizes d_i, $i = 1, 2, \ldots, n$. Then for each type of particle one can write an equation of the type (7), where the quantities D and u^s depend on the index i:

$$\frac{\partial p_i}{\partial t} = \frac{\partial}{\partial z}\left(D_i \frac{\partial p_i}{\partial z} - (u_i^s + v)\,p_i\right), \quad i = 1, 2, \ldots, n. \tag{8}$$

Then the coefficients D_i and u_i^s in (8) are functions of $c = \sum\limits_{i=1}^{n} c_i$. In order to obtain a closed system of equations from (8), we note that in correspondence with the law of large numbers there is the direct proportionality $c_i(z, t) \sim p_i(z, t)$. Thus (8) can be transformed to the system

$$\frac{\partial c_i}{\partial t} = \frac{\partial}{\partial z}\left(D_i \frac{\partial c_i}{\partial z} - (u_i^s + v)\,c_i\right), \quad i = 1, 2, \ldots, n. \tag{9}$$

We can express the velocity of motion of the fluid v in terms of the c_i. In order to do this we use the fact that if the suspension as a whole is at rest with respect to the walls of the apparatus, then we have the condition

$$(1-c)\,v + \sum_{i=1}^{n} c_i u_i = 0, \tag{10}$$

where $u_i = u_i^s + v$ is the mean velocity of motion of particles of the i-th kind with respect to the walls of the vessel. It follows from (10) that

$$v = -\sum_{i=1}^{n} c_i u_i^s(c). \tag{11}$$

We note that relation (11), and the formulas for the sedimentation velocities of narrow fractions in a homogeneous (c_i = const) suspension which follow from this relation have been experimentally verified in [7]. Substituting (11) into (9), we obtain a closed system of nonlinear parabolic equations for the volume concentration fields $c_i(z, t)$ of the fractions. The nonlinearity of the system (9) comes from the fact that its coefficients D_i, u_i^s, and v are functions of the c_i.

Up to now we have assumed that there is no source of the suspension and that the suspension does not move as a whole with respect to the walls of the apparatus. In the mathematical modelling of a continuously operating apparatus both of these effects must be taken into account. This is easily done (see [8]) by adding a term to the right-hand side of (9) describing the distribution density of sources of the suspension, and by adding the term $v_0(z)$ to $u_i^s + v$, where $v_0(z)$ is equal to the velocity of motion of the suspension, averaged over the cross section of the apparatus.

We consider the procedure for calculating in our model the concentration fields of the separate fractions for separation in a cylindrical apparatus whose axis is directed downwards. We take this axis to be the z axis and the origin is in the upper end of the cylinder. A polydisperse suspension is fed through a tube along the z axis, and the end of the tube is at the point z_0 ($0 < z_0 < H$). We neglect the diameter of the feeding tube, and therefore consider the source of the suspension to be a point source. The system of equations (9) for the cross-sectional averages of the concentrations of the narrow fractions can then be written in the form

$$\frac{\partial c_i}{\partial t} = \frac{\partial}{\partial z}\left(D_i \frac{\partial c_i}{\partial z} - (u_i^s(c) + v + v_0(z))\, c_i\right) + \frac{Q c_i^0}{\pi R^2}\, \delta(z - z_0) \tag{12}$$

(compare with [8], where the case of a monodisperse suspension was considered). From the balance condition with respect to the volume of the suspension we obtain for the velocity averaged over the cross section of motion of the suspension with respect to the walls of the vessel

$$v_0(z) = \begin{cases} Q_u/\pi R^2, & z_0 \leqslant z \leqslant H, \\ (Q - Q_u)/\pi R^2, & 0 \leqslant z < z_0. \end{cases} \tag{13}$$

We note that the "effective" diffusion coefficients in (12) can be quite different from (6) because of the contribution to the longitudinal mixing of the nonuniformity of the velocity of the suspension with respect to the cross section of the apparatus, i.e., its difference from $v_0(z)$; this is the so-called Taylor–Arisa diffusion [9].

In (12) it is convenient to transform to a dimensionless coordinate and time by dividing both sides by the characteristic sedimentation velocity $u^s = \left(\sum_{i=1}^{n} c_i^0\, u^s(0)\right)\left(\sum_{i=1}^{n} c_i^0\right)^{-1}$ and multiplying by H. The system (12) then takes the form

$$\frac{\partial c_i}{\partial \tau} = \frac{\partial}{\partial x}\left(\frac{1}{\mathrm{Pe}_i}\,\frac{\partial c_i}{\partial x} - j_i(c_1, c_2, \ldots, c_n)\right) + \beta c_i^0\, \delta(x - x_0), \tag{14}$$

where $x = z/H$; $\tau = u^s t/H$; $\mathrm{Pe}_i = u^s H/D_i$; $\beta = Q/\pi R^2 u^s$, and

$$j_i(c_1, c_2, \ldots, c_n) = \frac{c_i}{u^s}\left(u_i^s - \sum_{k=1}^{n} c_k u_k^s + v_0(x)\right) \tag{15}$$

is the dimensionless flux of the i-th fraction.

The system (14) must be supplemented by the initial (at $\tau = 0$) and boundary (at $x = 0$ and $x = 1$) conditions. As shown in [8], for the regime considered here one can take as the boundary conditions

$$\left.\frac{1}{\mathrm{Pe}_i}\,\frac{\partial c_i}{\partial x} - j_i\right|_{x=0} = c_i(0)\,(\beta - \alpha), \tag{16}$$

$$\frac{1}{\mathrm{Pe}_i}\frac{\partial c_i}{\partial x}-j_i\Big|_{x=1}=-c_i(1)\,\alpha, \tag{17}$$

where $\alpha=Q_u/\pi R^2u^s$. The conditions (16) and (17) mean that the total fluxes of each fraction on the upper and lower ends of the cylinder are equal to the amount of that fraction carried away per unit time by the suspension flowing through the corresponding end.

It is still necessary to specify the function $\Phi(c)$, which takes into account the hindrance of sedimentation. The well-known empirical formula of Richardson and Zaki has the form $\Phi(c)=(1-c)^{-4,65}$. A somewhat different dependence was used to compare the theoretical and and experimental results in [7], however it can be shown that this difference is quite insignificant for $c\leqslant 0.5$.

As an example of the use of the model given by (12), (16), and (17), we consider the concentration fields of large (i = 1) and small (i = 2) grains in the static regime for the case of a bifractional (n = 2) suspension feed. In the static regime the quantities α, β, c_1^0, c_2^0 are constants, and $\partial c_1/\partial\tau=\partial c_2/\partial\tau=0$. Under these conditions the system (12) with the boundary conditions (16) and (17) is easily reduced (see [8]) to the following nonlinear boundary-value problem for a system of two ordinary differential equations:

$$\frac{dc_i}{dx}=\mathrm{Pe}_i\left(j_i(c_1,\ c_2)+(\beta-\alpha)\,c_i(0)+\beta c_i^0\,\theta(x-x_0)\right), \tag{18}$$

$$c_i(0)(\beta-\alpha)+c_i(1)\,\alpha=\beta c_i^0,\ i=1,\ 2. \tag{19}$$

where θ in (18) denotes the Heaviside unit step function: $\theta(x)=0$ for $x\leqslant 0$ and $\theta(x)=1$ for $x>0$.

Equations (18) and (19) were solved numerically by reducing them to an initial-value problem. For a fixed pair of initial values $c_1(0)$ and $c_2(0)$, the Cauchy problem was solved for the system (18) with these initial values and the "errors" $\varepsilon_i=c_i(0)(\beta-\alpha)+c_i(1)\alpha-\beta c_i(0)$, i = 1, 2 were calculated. This procedure was continued until the quantities $|\varepsilon_i/\beta c_i^0|$ became less than a given value. We note that there is a simple physical interpretation of this condition: since (19) is the balance condition with respect to each of the fractions, the smallness of the error corresponds to a smallness of the "imbalance."

In the solution the following values were used for the constants (these correspond to conditions for gravitational separation in industrial potassium fertilizer): H = 3 m, R = 12 m, z_0 = 0.6 m, Q = 0.1 m^3/sec, Q_u = 0.02 m^3/sec, $d_1=5\cdot10^{-4}$ m, $d_2=10^{-4}$ m, D_1 = 0.02 m^2/sec, D_2 = 0.007 m^2/sec, $c_1^0=c_2^0=0.02$, $\rho_p=2.4\cdot10^3$ kg/m^3, $\rho_f=1.2\cdot10^3$ kg/m^3, $\mu=1.9\cdot10^{-3}$ Pa-sec. The allowed relative error (imbalance) for each of the fractions was taken to be 0.05.

From the calculation we obtained the static profiles of the volume concentrations of the fractions, as shown in Fig. 1. Curve 1 and 2 graphically demonstrate the significant difference between the two fractions. The concentration of the large grains monotonically increases with depth, whereas the dependence for the small grains has a maximum at x = 0.7. This is the point where the convective part of the flux is of the small-grain fraction changes sign, i.e., the small grains move downward on average for x < 0.7 and upward for x > 0.7. This behavior can be explained by the fact that when x > 0.7 there is a sudden increase in the volume concentration of the large grains moving downward and this causes an increase in the velocity of the displaced fluid upward, which in turn carries with it the small grains.

The available experimental data is such that the calculated profiles $c_i(x)$ cannot be compared to the actual ones (we note that for a monodisperse suspension such a comparison was given in [8]). Nevertheless, indirect experimental support for our results can be obtained as follows. The effectiveness of the separation is taken to be characterized by the change in the grain-size composition of the suspension of the top (at x = 0) and the bottom (at x = 1) in comparison with the composition of the suspension feed. In the example considered here, the calculated fractions of large and small grains in the feed was 0.5 and 0.5; at the top it was 0.84 and 0.16, respectively, and at the bottom it was 0.005 and 0.995, respectively. If we represent a real suspension feed as bifractional with different contents of large and small grains, then experimentally the following fractions are obtained: 0.76 and 0.24 at the top and 0.009 and 0.991 at the bottom, and these values are in satisfactory agreement with the theoretical values.

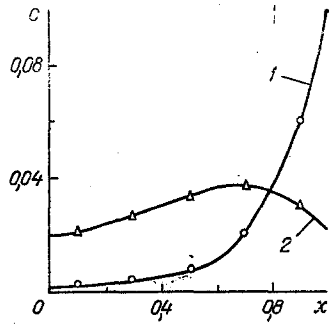

Fig. 1. Static profiles of the volume contents of large (curve 1) and small (curve 2) grains in the continuous separation regime.

Finally we note that although the general structure of our model is rather firmly based on accepted theoretical premises, the concrete expressions for the convective fluxes and particularly for the coefficients of diffusion are subject to further refinement. Our results show that the model obtained here can be useful in describing realistic processes even with the simplest assumptions such as the independence of the coefficients of diffusion on c. In this approach, a "theoretical" dependence of the type (6) is used only to estimate the order of magnitude of these coefficients, and their final values are selected by fitting the parameters of the model to experiments.

NOTATION

c, c_i^0, volume content of the solid phase in the apparatus and feed; d, linear dimension of the solid particles, m; D, coefficient of diffusion, m^2/sec; i, index enumerating the narrow fractions of the solid phase; K, normalized correlation function of the concentration fluctuations of the solid phase; H, R, height and radius of the apparatus, m; u, v, w, velocity of the solid phase, fluid and relative velocity, m/sec; Q, Q_u, volume flow rate of the suspension in the feed and underflow, m^3/sec; z, coordinate along the axis of the apparatus, m; μ, viscosity of the fluid phase, Pa-sec; ρ_p, ρ_f, density of the solid particles and density of the fluid, kg/m^3; σ^2, variance of the concentration fluctuations of the solid phase; ξ(t) standard white noise with unit intensity.

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